

Cash-settled European Swaptions

The standard market formula used to price cash-settled swaptions is a copy of the physical delivery Black formula, where the classical annuity term is replaced with a single-factor one (discounting on the underlying swap rate fixing at maturity).

As reported in Mercurio and OpenGamma notes, this formula is not arbitrage-free. In order to get a proper pricing, one has to take into account the convexity between the swap rate and the ratio cash settled vs. physical annuity. Using a one factor LGM model, we derive a formula for this convexity.

With this new formula, put-call parity doesn't apply anymore for cash-settled swaptions, that is to say a 0-wide collar price may not be zero (with call and put strikes equal to the forward swap rate).

We assess here the differences between the standard market formula and the new one for European swaptions: ATM straddles, 0-wide collars, 10y-underlying-swap collars and 30y-underlying-swap collars.

ATM straddles: new formula – standard formula

Expiry	1Y	2Y	3Y	4Y	5Y	7Y	10Y	15Y	20Y	30Y
▶ 1M	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp
3M	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp
6M	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.3 bp
1Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.7 bp
2Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.3 bp	0.7 bp	1.8 bp
3Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.6 bp	1.2 bp	3.0 bp
4Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.3 bp	0.9 bp	1.8 bp	4.3 bp
5Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.5 bp	1.2 bp	2.3 bp	5.6 bp
7Y	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.1 bp	0.3 bp	0.7 bp	1.9 bp	3.4 bp	8.2 bp
10Y	0.0 bp	0.0 bp	0.1 bp	0.1 bp	0.2 bp	0.5 bp	1.2 bp	2.9 bp	5.0 bp	11.8 bp
20Y	0.0 bp	0.1 bp	0.2 bp	0.3 bp	0.4 bp	0.9 bp	2.0 bp	4.7 bp	8.2 bp	17.5 bp
30Y	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.4 bp	0.8 bp	1.9 bp	4.8 bp	8.8 bp	17.8 bp

0-wide collars: new formula

Expiry	1Y	2Y	3Y	4Y	5Y	7Y	10Y	15Y	20Y	30Y
1M	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.3 bp	0.7 bp
3M	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.4 bp	0.8 bp	2.0 bp
6M	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.7 bp	1.4 bp	4.2 bp
1Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.4 bp	1.3 bp	2.7 bp	7.8 bp
2Y	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.3 bp	0.9 bp	2.8 bp	5.7 bp	15.0 bp
3Y	0.0 bp	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.5 bp	1.6 bp	4.4 bp	8.8 bp	21.5 bp
4Y	0.0 bp	0.0 bp	0.1 bp	0.1 bp	0.3 bp	0.8 bp	2.1 bp	5.9 bp	11.5 bp	27.8 bp
5Y	0.0 bp	0.0 bp	0.1 bp	0.2 bp	0.4 bp	1.0 bp	2.7 bp	7.1 bp	13.8 bp	33.2 bp
▶ 7Y	0.0 bp	0.1 bp	0.1 bp	0.4 bp	0.6 bp	1.5 bp	3.7 bp	9.7 bp	18.1 bp	43.0 bp
10Y	0.1 bp	0.1 bp	0.3 bp	0.6 bp	1.0 bp	2.3 bp	5.2 bp	13.0 bp	23.1 bp	54.2 bp
20Y	0.0 bp	0.2 bp	0.5 bp	0.8 bp	1.3 bp	2.8 bp	6.3 bp	15.3 bp	27.6 bp	61.5 bp
30Y	0.0 bp	0.1 bp	0.3 bp	0.6 bp	1.1 bp	2.2 bp	5.3 bp	13.4 bp	25.6 bp	54.6 bp

10Y underlying swap collars: new formula – standard formula

Expiry	Forward	Straddle	0	25	50	100	150	200	300	400
▶ 3M	0.7654 %	0.0 bp	0.1 bp	0.1 bp	0.1 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp	0.0 bp
1Y	0.9155 %	0.0 bp	0.4 bp	0.4 bp	0.3 bp	0.2 bp	0.1 bp	0.1 bp	0.0 bp	0.0 bp
2Y	1.1194 %	0.1 bp	0.9 bp	0.8 bp	0.7 bp	0.6 bp	0.4 bp	0.3 bp	0.1 bp	0.1 bp
3Y	1.3099 %	0.2 bp	1.6 bp	1.4 bp	1.3 bp	1.1 bp	0.8 bp	0.7 bp	0.4 bp	0.2 bp
5Y	1.6105 %	0.5 bp	2.7 bp	2.5 bp	2.3 bp	2.0 bp	1.7 bp	1.4 bp	1.0 bp	0.6 bp
7Y	1.7783 %	0.7 bp	3.7 bp	3.5 bp	3.3 bp	2.9 bp	2.6 bp	2.2 bp	1.6 bp	1.1 bp
10Y	1.8080 %	1.2 bp	5.2 bp	4.9 bp	4.7 bp	4.3 bp	3.8 bp	3.4 bp	2.6 bp	2.0 bp
15Y	1.5834 %	1.9 bp	6.6 bp	6.3 bp	6.1 bp	5.6 bp	5.1 bp	4.6 bp	3.7 bp	3.0 bp
20Y	1.3852 %	2.0 bp	6.3 bp	6.1 bp	5.8 bp	5.4 bp	5.0 bp	4.5 bp	3.7 bp	3.0 bp
30Y	1.0378 %	1.9 bp	5.3 bp	5.1 bp	4.9 bp	4.6 bp	4.2 bp	3.9 bp	3.2 bp	2.6 bp

30Y underlying swap collars: new formula – standard formula

Expiry	Forward	Straddle	0	25	50	100	150	200	300	400
▶ 3M	1.2878 %	0.1 bp	2.0 bp	1.5 bp	1.0 bp	0.4 bp	0.1 bp	0.0 bp	0.0 bp	0.0 bp
1Y	1.3274 %	0.7 bp	7.8 bp	6.7 bp	5.7 bp	3.8 bp	2.4 bp	1.5 bp	0.5 bp	0.2 bp
2Y	1.3795 %	1.8 bp	15.0 bp	13.5 bp	12.0 bp	9.2 bp	6.8 bp	4.9 bp	2.4 bp	1.1 bp
3Y	1.4253 %	3.0 bp	21.5 bp	19.7 bp	17.9 bp	14.4 bp	11.4 bp	8.7 bp	4.9 bp	2.7 bp
5Y	1.4874 %	5.6 bp	33.2 bp	31.0 bp	28.7 bp	24.3 bp	20.2 bp	16.6 bp	10.6 bp	6.6 bp
7Y	1.5028 %	8.2 bp	43.0 bp	40.4 bp	37.9 bp	32.8 bp	28.1 bp	23.6 bp	16.2 bp	10.7 bp
10Y	1.4466 %	11.8 bp	54.2 bp	51.3 bp	48.5 bp	42.9 bp	37.5 bp	32.3 bp	23.3 bp	16.3 bp
15Y	1.2590 %	15.6 bp	61.0 bp	58.2 bp	55.3 bp	49.7 bp	44.2 bp	38.9 bp	29.5 bp	21.7 bp
20Y	1.0922 %	17.5 bp	61.5 bp	58.7 bp	56.0 bp	50.7 bp	45.4 bp	40.4 bp	31.2 bp	23.5 bp
30Y	0.8222 %	17.8 bp	54.6 bp	52.3 bp	50.1 bp	45.6 bp	41.2 bp	37.0 bp	29.2 bp	22.6 bp

References:

- http://www.fabiomercurio.it/cashsettled_note.pdf
- <https://developers.opengamma.com/quantitative-research/Swaption-Pricing-OpenGamma.pdf>